OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE **4724/01**

MATHEMATICS

Core Mathematics 4

TUESDAY 22 JANUARY 2008 Afternoon

Time: 1 hour 30 minutes

Additional materials: Answer Booklet (8 pages)
List of Formulae (MF1)

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Read each question carefully and make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- **You are reminded of the need for clear presentation in your answers.**

This document consists of **4** printed pages.

[Turn over

1 Find the angle between the vectors $\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$ and $2\mathbf{i} + \mathbf{j} + \mathbf{k}$. [4]

2 (i) Express $\dfrac{x}{(x+1)(x+2)}$ in partial fractions. [3]

(ii) Hence find $\displaystyle\int \frac{x}{(x+1)(x+2)}\,\mathrm{d}x$. [2]

3 When $x^4 - 2x^3 - 7x^2 + 7x + a$ is divided by $x^2 + 2x - 1$, the quotient is $x^2 + bx + 2$ and the remainder is $cx + 7$. Find the values of the constants a, b and c. [5]

4 Find the equation of the normal to the curve

$$x^3 + 4x^2y + y^3 = 6$$

at the point (1, 1), giving your answer in the form $ax + by + c = 0$, where a, b and c are integers. [6]

5 The vector equations of two lines are

$$\mathbf{r} = (5\mathbf{i} - 2\mathbf{j} - 2\mathbf{k}) + s(3\mathbf{i} - 4\mathbf{j} + 2\mathbf{k}) \quad \text{and} \quad \mathbf{r} = (2\mathbf{i} - 2\mathbf{j} + 7\mathbf{k}) + t(2\mathbf{i} - \mathbf{j} - 5\mathbf{k}).$$

Prove that the two lines are

(i) perpendicular, [3]

(ii) skew. [5]

6 (i) Expand $(1 + ax)^{-4}$ in ascending powers of x, up to and including the term in x^2. [3]

(ii) The coefficients of x and x^2 in the expansion of $(1 + bx)(1 + ax)^{-4}$ are 1 and -2 respectively. Given that $a > 0$, find the values of a and b. [5]

7 (i) Given that

$$A(\sin\theta + \cos\theta) + B(\cos\theta - \sin\theta) \equiv 4\sin\theta,$$

find the values of the constants A and B. [3]

(ii) Hence find the exact value of

$$\int_0^{\frac{1}{4}\pi} \frac{4\sin\theta}{\sin\theta + \cos\theta}\,\mathrm{d}\theta,$$

giving your answer in the form $a\pi - \ln b$. [5]

8 Water flows out of a tank through a hole in the bottom and, at time t minutes, the depth of water in the tank is x metres. At any instant, the rate at which the depth of water in the tank is decreasing is proportional to the square root of the depth of water in the tank.

(i) Write down a differential equation which models this situation. [2]

(ii) When $t = 0, x = 2$; when $t = 5, x = 1$. Find t when $x = 0.5$, giving your answer correct to 1 decimal place. [6]

9 The parametric equations of a curve are $x = t^3, y = t^2$.

(i) Show that the equation of the tangent at the point P where $t = p$ is

$$3py - 2x = p^3.$$ [4]

(ii) Given that this tangent passes through the point $(-10, 7)$, find the coordinates of each of the three possible positions of P. [5]

10 **(i)** Use the substitution $x = \sin\theta$ to find the exact value of

$$\int_0^{\frac{1}{2}} \frac{1}{(1-x^2)^{\frac{3}{2}}}\,\mathrm{d}x.$$ [6]

(ii) Find the exact value of

$$\int_1^3 \frac{\ln x}{x^2}\,\mathrm{d}x.$$ [5]

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1	Method for finding magnitude of any vector	M1		Expect $\sqrt{14}$ and $\sqrt{6}$
	Method for finding scalar prod of any 2 vectors	M1		Expect $1.2+(-2).1+3.1=3$
	Using $\cos\theta = \dfrac{\mathbf{i}-2\mathbf{j}+3\mathbf{k}\,.\,2\mathbf{i}+\mathbf{j}+\mathbf{k}}{\lvert\mathbf{i}-2\mathbf{j}+3\mathbf{k}\rvert\lvert2\mathbf{i}+\mathbf{j}+\mathbf{k}\rvert}$	M1		Correct vectors only. Expect $\cos\theta = \dfrac{3}{\sqrt{14}\sqrt{6}}$
	70.9 (70.89, 70.893) WWW; 1.24 (1.237)	A1	**4**	Condone answer to nearest degree (71)
2	**(i)** Correct format $\dfrac{A}{x+1}+\dfrac{B}{x+2}$	M1		stated or implied by answer
	$-\dfrac{1}{x+1}$ or $A=-1$	A1		
	$+\dfrac{2}{x+2}$ or $B=2$	A1	**3**	
	(ii) $\int \dfrac{1}{x+1}\,dx = \ln(x+1)$ or $\ln\lvert x+1\rvert$			
	or $\int \dfrac{1}{x+2}\,dx = \ln(x+2)$ or $\ln\lvert x+2\rvert$	B1		
	$A\ln\lvert x+1\rvert + B\ln\lvert x+2\rvert + c$ ISW	√A1	**2**	Expect $-\ln\lvert x+1\rvert + 2\ln\lvert x+2\rvert + c$
3	Method 1 (Long division)			
	Clear correct division method at beginning	M1		x^2 in quot, mult back & attempt subtraction [At subtraction stage, cf $(x^4)=0$]
	Correct method up to & including x term in quot	M1		[At subtraction stage, cf $(x^3)=0$]
	Method 2 (Identity)			
	Writing $(x^2+2x-1)(x^2+bx+2)+cx+7$	M1		Probably equated to $x^4-2x^3-7x^2+7x+a$
	Attempt to compare cfs of x^3 or x^2 or x or const	M1		
	Then:			
	$b=-4$	A1		
	$c=-1$	A1		
	$a=5$	A1	**5**	
4	$\dfrac{d}{dx}(x^2y) = x^2\dfrac{dy}{dx}+2xy$	B1		s.o.i.;
	$\dfrac{d}{dx}(y^3) = 3y^2\dfrac{dy}{dx}$	B1		
	Substitute $(x,y)=(1,1)$ and solve for $\dfrac{dy}{dx}$	M1		or v.v. Solve now or at normal stage. [This
		M1		dep on either/both B1 earned]
	$\dfrac{dy}{dx} = -\dfrac{11}{7}$ WWW	A1		Implied if grad normal = $\dfrac{7}{11}$
	Gradient normal = $-\dfrac{1}{\frac{dy}{dx}}$	M1		Numerical or general, awarded at any stage
	$7x-11y+4=0$ AEF	A1	**6**	No fractions in final answer.

5	**(i)** Use $3\mathbf{i}-4\mathbf{j}+2\mathbf{k}$ and $2\mathbf{i}-\mathbf{j}-5\mathbf{k}$ only	M1		
	Use correct method for scalar prod of <u>any</u> 2 vectors	M1		(indep) May be as part of $\cos\theta = \frac{a.b}{\lvert a\rVert b\rvert}$
	Obtain $6+4-10$, state = 0 & deduce perp **AG**	A1	**3**	
	(ii) Produce 3 equations in s and t	*M1		of the type $5+3s=2+2t$, $-2-4s=-2-t$ and $-2+2s=7-5t$
	Solve 2 of the equations for s and t	dep*M1		*<u>Or</u> Eliminate s (or t) from 2 pairs dep*M1*
	Obtain $(s,t)=\left(\frac{3}{5},\frac{12}{5}\right)$ or $\left(\frac{9}{22},\frac{18}{11}\right)$ or $\left(\frac{3}{19},\frac{33}{19}\right)$	A1		*(5t=12,11t=18,19t=33) <u>or</u> (5s=3,22s= 9,19s=3) A1,A1*
	Substitute their values in 3rd equation	dep*M1		*State/show inconsistency <u>& state non-parallel</u>*
	State/show inconsistency <u>& state non-parallel</u> ∴ skew	A1	**5**	*∴ skew WWW* A1
6	**(i)** $1-4ax+\ldots$	B1		
	$\frac{-4.-5}{1.2}(ax)^2$ or $\frac{-4.-5}{1.2}a^2x^2$ or $\frac{-4.-5}{1.2}ax^2$	M1		Do not accept $\begin{pmatrix}-4\\2\end{pmatrix}$ unless 10 also appears
	$\ldots+10a^2x^2$	A1	**3**	
	(ii) f.t. (their cf x) + b(their const cf) = 1	√B1		Expect $b-4a=1$
	f.t. (their cf x^2) + b(their cf x) $=-2$	√B1		Expect $10a^2-4ab=-2$
	Attempt to eliminate 'b' and produce equation in 'a'	M1		Or eliminate 'a' and produce equation in 'b'
	Produce $6a^2+4a=2$ AEF	A1		Or $6b^2+4b=42$ AEF
	$a=\frac{1}{3}$ and $b=\frac{7}{3}$ only	A1	**5**	Made clear to be only (final) answer
7	**(i)** Perform an operation to produce an equation connecting A and B (or possibly in A or in B)	M1		Probably substituting value of θ, or comparing coefficients of sin x, and/or cos x
	$A=2$	A1		
	$B=-2$	A1	**3**	WW scores 3
	(ii) Write $4\sin\theta$ as $A(\sin\theta+\cos\theta)+B(\cos\theta-\sin\theta)$ and re-write integrand as $A+\frac{B(\cos\theta-\sin\theta)}{\sin\theta+\cos\theta}$	M1		*A* and *B* need not be numerical – but, if they are, they should be the values found in (i).
	$\int A\,\mathrm{d}\theta = A\theta$	√B1		general or numerical
	$\int \frac{B(\cos\theta-\sin\theta)}{\sin\theta+\cos\theta}\mathrm{d}\theta = B\ln(\sin\theta+\cos\theta)$	√A2		general or numerical
	Produce $\frac{1}{4}A\pi+B\ln\sqrt{2}$ f.t. with their A,B	√A1	**5**	Expect $\frac{1}{2}\pi-\ln 2$ (Numerical answer only)
8	**(i)** $\frac{\mathrm{d}x}{\mathrm{d}t}$ or $-kx^{\frac{1}{2}}$ or $kx^{\frac{1}{2}}$ seen	M1		*k* non-numerical; i.e. 1 side correct
	$\frac{\mathrm{d}x}{\mathrm{d}t}=-kx^{\frac{1}{2}}$ or $\frac{\mathrm{d}x}{\mathrm{d}t}=kx^{\frac{1}{2}}$	A1	**2**	i.e. both sides correct
	(ii) Separate variables or invert, + attempt to integrate *	M1		Based <u>only</u> on above eqns or $\frac{\mathrm{d}x}{\mathrm{d}t}=x^{\frac{1}{2}}$, $-x^{\frac{1}{2}}$
	Correct result for their equation after integration	A1		Other than omission of 'c'
	Subst $(t,x)=(0,2)$ into eqn containing k &/or c	dep*M1		or substitute (5,1)
	Subst $(t,x)=(5,1)$ into eqn containing k <u>&</u> c	dep*M1		or substitute (0,2)
	Subst $x=0.5$ into eqn with their k & c subst	dep*M1		
	$t=8.5\,(8.5355339)$	A1	**6**	[1 d.p. requested in question]

9	**(i)** Use $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$ or $\frac{\frac{dy}{dp}}{\frac{dx}{dp}}$	M1		Or conv to cartes form & att to find $\frac{dy}{dx}$ at P
	$= \frac{2t}{3t^2}$ or $\frac{2p}{3p^2}$	A1		
	Find eqn tgt thro (p^3, p^2) or (t^3, t^2) ,their gradient	M1		Using $y - y_1 = m(x - x_1)$ or $y = mx + c$
	$3py - 2x = p^3$ **AG**	A1	**4**	Do not accept t here
	(ii) Substitute $(-10,7)$ into given equation *	M1		to produce a cubic equation in p
	Satis attempt to find at least 1 root/factor dep*	M1		Inspection/factor theorem/rem theorem/t&i
	Any one root	A1		-1 or -4 or 5
	All 3 roots	A1		-1,-4 and 5
	$(-1,1)$, $(-64,16)$ and $(125,25)$	A1	**5**	All 3 sets; no f.t.
10	**(i)** $(1-x^2)^{\frac{3}{2}} \rightarrow \cos^3\theta$	B1		May be implied by $\int \sec^2\theta \, d\theta$
	$dx \rightarrow \cos\theta \, d\theta$	B1		
	$\frac{1}{(1-x^2)^{\frac{3}{2}}} dx \rightarrow \sec^2\theta (d\theta)$ or $\frac{1}{\cos^2\theta}(d\theta)$	B1		
	$\int \sec^2\theta (d\theta) = \tan\theta$	B1		
	Attempt change of limits (expect 0 & $\frac{1}{6}\pi / 30$)	M1		Use with $f(\theta)$; or re-subst & use 0 & $\frac{1}{2}$
	$\frac{1}{\sqrt{3}}$ AEF	A1	**6**	Obtained with no mention of 30 anywhere
	(ii) Use parts with $u = \ln x$, $\frac{dv}{dx} = \frac{1}{x^2}$	*M1		obtaining a result $f(x) + / - \int g(x)(dx)$
	$-\frac{1}{x}\ln x + \int \frac{1}{x^2}(dx)$ AEF	A1		Correct first stage result
	$-\frac{1}{x}\ln x - \frac{1}{x}$	A1		Correct overall result
	Limits used correctly	dep*M1		
	$\frac{2}{3} - \frac{1}{3}\ln 3$	A1	**5**	
	<u>If substitution attempted in part (ii)</u>			
	$\ln x = t$	B1		
	Reduces to $\int t\, e^{-t} \, dt$	B1		
	Parts with $u = t, dv = e^{-t}$	M1		
	$-te^{-t} - e^{-t}$	A1		
	$\frac{2}{3} - \frac{1}{3}\ln 3$	A1		